Experiment can decide between the standard and Bohmian quantum mechanics

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Abstract

In this investigation, we suggest a thought experiment to make a comparison between predictions of standard and Bohmian quantum mechanics. We consider a two-particle device at two various situations of the entangled and unentangled systems. In the first case, the two theories can predict different results at the individual level, while their statistical results are the same. In the other case, not only they are in disagreement at the individual level, but also their equivalence breaks down at the statistical level, if one uses selective detection. Furthermore, we discuss about some objections that can be raised concerning the results of the experiment.

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1 Introduction

The statistical interpretation of the wave function of the standard quantum mechanics (SQM) is consistent with all performed experiments. An interference pattern on a screen is built up by a series of apparently random events, and the wave function correctly predicts where the particle is most likely to land in an ensemble of trials. Instead, one may take the view that the characteristic distribution of spots on a screen which build up an interference pattern is an evidence that the wave function has a more potent physical role. If one attempts to understand the experimental results as the outcome of a causally connected series of individual process, then one is free to inquire further significance of the wave function and to introduce other concepts in addition to the wave function. Bohm [1] in 1952 showed that an individual physical system comprises a wave propagating in space-time together with a point particle which moves continuously under the guidence of the wave [1-4]. He applied his theory to a range of examples drawn from non-relativistic quantum mechanics and speculated on possible alternations in the particle and field laws of motion such that the predictions of the modified theory continue to agree with those of SQM, where this is tested but could disagree in as yet unexplored domains [3]. For instance, when Bohm presented his theory in 1952, experiments could be done with an almost continuous beam of particles. Thus, it is impossible to discriminate between the standard and Bohmian quantum mechanics (BQM). The two theories can be discriminated at the individual level, because SQM is a probabilistic theory while BQM is a precisely defined and deterministic one.

In recent years, the significance of proposals that can predict different results between SQM and BQM have been the subject of many discussions [for example, 5-17]. At first, it seems that definition of time spent by a particle within a classically forbidden barrier provides a good evidence for the preference of BQM. But, there are difficult technical questions, both theoretically and experimentally, that are still unsolved about these tunneling times [4, 5]. On the other hand, Englert et al. [6] and Scully [7] have claimed that in some cases the Bohm approach gives results that disagree with those obtained from SQM and, in consequence, with experiment. Concerning this, at first Dewdney et al. [8] and then Hiley et al. [9] showed that the specific objections raised by Englert and Scully cannot be sustained. Furthermore, Hiley believe that no experiment can decide between the standard interpretation and Bohm's interpretation. However, Vigier [10], in his recent work, has given a brief list of new experiments which suggest that the U(1) invariant massless photon assumed properties of light within the standard interpretation, are too restrictive and that the O(3) invariant massive photon causal de Broglie-Bohm interpretation of quantum mechanics, is now supported by experiments. In addition, Leggett [11] considered some thought experiments involving macrosystems which can predict different results concerning SQM and BQM[1]. Furthermore, in some of the recent works, feasible thought experiments have been suggested to distinguish between SQM and BQM [12, 13]. In one of the works, Ghose [12] indicated that although BQM is equivalent to SQM when averages of dynamical variables are taken over a Gibbs ensemble of Bohmian trajectories, the equivalence breaks down for ensembles built over clearly separated short intervals of time in special entangled two-bosonic particle systems. In the other work [13], we extended the Ghose's

[1]He assumed that the experimental predictions of SQM will continue to be realized under the extreme conditions specified, although a test of this hypothesis is part of the aim of the macroscopic quantum cohrence program. In addition, he considered BQM as an interpretation theory rather than an alternative theory.

work using a different Gaussian wave function. There, we obtained statistical disagreement between the two theories, using selective detection. Marchildon [14, 15] has tried to show that there is no reason to expect discrepancies between BQM and SQM in the content of the two-particle interference devices. Furthermore, in our private communications, we received some objections about statistical disagreement for unentangled wave functions, source deviation and selective detection. Ghose [16] claims that Marchildon's arguments against his work are refuted and his basic conclusion stands. In addition, we have shown elsewhere [17] that our statistical disagreement is also valid if one applies the two entangled particles without source deviation from the axis of symmetry, but using another kind of selective detection. Furthermore, the role of selective detection to alter the statistics of predictions is shown by Durr et al. [18].

In this work, we study two kinds of Gaussian wave function that can be considered for a two-particle interference device. SQM and BQM predictions are compared at both the individual and the statistical level[2]. We also discuss about some objections that are raised.

2 Description of two particles double-slit experiment

Consider the famous double-slit experiment. Instead of the usual one particle emitting source, one can consider a special point source S_1 so that a pair of correlated identical non-relativistic particles originate simultaneously from it. We assume that the intensity of the beam is so low that a time we have only a single pair of particles passing through the slits. If we assume that the detection screen S_2 registers only those pairs of particles that reach it simultaneously, the interference effects of single particles will be eliminated. Furthermore, it is assumed that the detection process has no causal role in the phenomenon of interference [3]. In two dimensional coordinate system, with the origin at O, the centers of the two slits are located at $(0, \pm Y)$. Figure 1 shows general scheme of this two-slit experiment.

Concerning the assumed source, we can have two alternatives:
1) The wave function of two emitted particles are entangled. In the other words, if one particle passes from the upper (lower) slit, the other particle must go through lower (upper) slit. we can also say that the total momentum of two particles in the y-direction is zero.
2) Wave function of two emitted particles have no correlation and they are unentangled. In the other words, the emission of one particle is done freely and two particles can be considered independent.
In addition, we assume that the centre of mass of the two particles are adjustable. This assumption imposes a correlation between the two particles in both cases. In the following we want to examine each one of the two cases, separately, and apply them using SQM.

2.1 Entangled wave function

We take the incident wave on the double-slit screen to be a plane wave of the form

$$\psi_{in}(x_1, y_1; x_2, y_2; t) = ae^{i[k_x(x_1+x_2)+k_y(y_1-y_2)]}e^{-iEt/\hbar}, \tag{1}$$

where a is a constant and $E = E_1 + E_2 = \hbar^2(k_x^2 + k_y^2)/m$ is the total energy of the system of two identical particles. m is the mass of each particle and k_i is the wave number of particle

[2] The individual level refers to our experiment with a pair of particles which are emitted in clearly separated short intervals of time, and by statistical level we mean our final interference pattern.

in the i-direction. For mathematical simplicity we avoid slits with sharp edges which produce mathematical complexity of Fresnel diffraction, i.e., we assume that the slits have soft edges, so that the Gaussian wave packets are produced along the y-direction, and that the plane wave along the x-axis remain unchanged [3]. We take the time of the formation of the Gaussian wave to be $t = 0$. Then, the emerging wave packets from the slits A and B are respectively

$$\psi_A(x, y) = a(2\pi\sigma_0^2)^{-1/4} e^{-(y-Y)^2/4\sigma_0^2} e^{i[k_x x + k_y(y-Y)]}, \tag{2}$$

$$\psi_B(x, y) = a(2\pi\sigma_0^2)^{-1/4} e^{-(y+Y)^2/4\sigma_0^2} e^{i[k_x x - k_y(y+Y)]}, \tag{3}$$

where σ_0 is the half-width of each slit.

Now, for this two–particle system, the total wave function at the detection screen S_2, at time t, can be written as

$$\psi(x_1, y_1; x_2, y_2; t) = \frac{1}{\sqrt{2}}[\psi_A(x_1, y_1, t)\psi_B(x_2, y_2, t) \pm \psi_A(x_2, y_2, t)\psi_B(x_1, y_1, t)], \tag{4}$$

with

$$\psi_A(x, y, t) = a(2\pi\sigma_t^2)^{-1/4} e^{-(y-Y-u_y t)^2/4\sigma_0\sigma_t} e^{i[k_x x + k_y(y-Y-u_y t/2) - E_x t/\hbar]}, \tag{5}$$

$$\psi_B(x, y, t) = a(2\pi\sigma_t^2)^{-1/4} e^{-(y+Y+u_y t)^2/4\sigma_0\sigma_t} e^{i[k_x x - k_y(y+Y+u_y t/2) - E_x t/\hbar]}, \tag{6}$$

where

$$\sigma_t = \sigma_0(1 + \frac{i\hbar t}{2m\sigma_0^2}), \tag{7}$$

$$u_y = \frac{\hbar k_y}{m}; E_x = \frac{1}{2} m u_x^2. \tag{8}$$

Note that the upper and lower signs in the total entangled wave function (4) are due to symmetric and anti-symmetric wave functions under the exchange of particles 1 and 2, corresponding to bosonic and fermionic property, respectively.

2.2 Unentangled wave function

In this case, the incident plane wave can be considered to be

$$\tilde{\psi}_{in}(x_1, y_1; x_2, y_2; t) = a e^{i[k_x(x_1+x_2) + k_y(\pm y_1 \pm y_2)]} e^{-iEt/\hbar}, \tag{9}$$

where it contains four terms corresponding to each sign. Now, for this two-particle system, the total wave function at time t can be written as

$$\begin{aligned}
&\tilde{\psi}(x_1, y_1; x_2, y_2; t) = \\
&\quad N[\psi_A(x_1, y_1, t)\psi_B(x_2, y_2, t) + \psi_A(x_2, y_2, t)\psi_B(x_1, y_1, t) \\
&\quad +\psi_A(x_1, y_1, t)\psi_A(x_2, y_2, t) + \psi_B(x_1, y_1, t)\psi_B(x_2, y_2, t)] \\
&\quad = N[\psi_A(x_1, y_1, t) + \psi_B(x_1, y_1, t)][\psi_A(x_2, y_2, t) + \psi_B(x_2, y_2, t)],
\end{aligned} \tag{10}$$

where N is a reparameterization constant and its value is unimportant in this work.

2.3 SQM prediction

Based on SQM, the wave function can be associated with an individual physical system. It provides the most complete description of the system that is, in principle, possible. The nature of description is statistical, and concerns the probabilities of the outcomes of all conceivable measurements that may be performed on the system. It is well-known from SQM that the probability of simultaneous detection of the particles at y_M and y_N, at the screen S_2, located at $x_1 = x_2 = D$, and $t = D/u_x$, is equal to

$$P_{12}(y_M, y_N) = \int_{y_M}^{y_M+\triangle} dy_1 \int_{y_N}^{y_N+\triangle} dy_2 |\psi(x_1, y_1; x_2, y_2; t)|^2. \tag{11}$$

The parameter Δ, which is taken to be small, is a measure of the size of the detectors. We shall compare this prediction of SQM with that of BQM.

3 Bohmian predictions and their comparison with SQM

Based on basic postulates of BQM, an individual physical system comprises a wave propagating in space-time together with a point particle which moves continuosly under the guidance of the wave. The wave function $\psi(\overrightarrow{x}, t)$ is a solution of Schrödinger's equation and the particle motion is obtained from the following first order differential equation

$$\dot{\overrightarrow{x_i}}(\overrightarrow{x}, t) = \frac{1}{m_i} \overrightarrow{\nabla}_i S(\overrightarrow{x}, t) = \frac{\hbar}{m_i} Im \left(\frac{\overrightarrow{\nabla}_i \psi(\overrightarrow{x}, t)}{\psi(\overrightarrow{x}, t)} \right), \tag{12}$$

where $\overrightarrow{x} = (\overrightarrow{x_1}, \overrightarrow{x_2}, ..., \overrightarrow{x_n})$, and $S(\overrightarrow{x}, t)$ is the phase of $\psi(\overrightarrow{x}, t)$ in polar form, that is,

$$\psi(\overrightarrow{x}; t) = R(\overrightarrow{x}; t) e^{iS(\overrightarrow{x}; t)/\hbar}, \tag{13}$$

To compare between the two theories, here, we study the speed of particles 1 and 2 in the y-direction, that is

$$\dot{y}_1(x_1, y_1; x_2, y_2; t) = \frac{\hbar}{m} Im \frac{\partial_{y_1} \psi(x_1, y_1; x_2, y_2; t)}{\psi(x_1, y_1; x_2, y_2; t)}, \tag{14}$$

$$\dot{y}_2(x_1, y_1; x_2, y_2; t) = \frac{\hbar}{m} Im \frac{\partial_{y_2} \psi(x_1, y_1; x_2, y_2; t)}{\psi(x_1, y_1; x_2, y_2; t)}. \tag{15}$$

We remember that here two kinds of the wave function could be considered, entangled and unentangled. Thus, in the following we study each of them, separately.

3.1 Predictions for entangled wave function

Consider the entangled wave function (4). Replacing of it by (14) and (15), we have

$$\begin{aligned} \dot{y}_1 &= \frac{\hbar}{\sqrt{2}m} Im \frac{1}{\psi} [[-2(y_1 - Y - u_y t)/4\sigma_0\sigma_t + ik_y]\psi_{A_1}\psi_{B_2} \\ &\pm [-2(y_1 + Y + u_y t)/4\sigma_0\sigma_t - ik_y]\psi_{A_2}\psi_{B_1}], \end{aligned} \tag{16}$$

$$\begin{aligned}\dot{y}_2 &= \frac{\hbar}{\sqrt{2}m} Im\frac{1}{\psi}[[-2(y_2+Y+u_yt)/4\sigma_0\sigma_t - ik_y]\psi_{A_1}\psi_{B_2} \\ &\pm [-2(y_2-Y-u_yt)/4\sigma_0\sigma_t + ik_y]\psi_{A_2}\psi_{B_1}],\end{aligned} \tag{17}$$

On the other hand, from (5) and (6) one can see that

$$\begin{aligned}\psi_A(x_1,y_1,t) = \psi_B(x_1,-y_1,t),\\ \psi_A(x_2,y_2,t) = \psi_B(x_2,-y_2,t),\end{aligned} \tag{18}$$

which indicates the reflection symmetry of $\psi(x_1,y_1;x_2,y_2;t)$ with respect to the x–axis. Using this symmetry in (16) and (17) we have

$$\begin{aligned}\dot{y}_1(x_1,y_1;x_2,y_2;t) = \mp\dot{y}_1(x_1,-y_1;x_2,-y_2;t),\\ \dot{y}_2(x_1,y_1;x_2,y_2;t) = \mp\dot{y}_2(x_1,-y_1;x_2,-y_2;t).\end{aligned} \tag{19}$$

These relations show that if $y_1(t) = y_2(t) = 0$ i.e., two particles are on the x-axis simultaneously, then the speed of each bosonic particles in the y-direction is zero along the symmetry axis x, but we have no such constraint on fermionic particles. We have shown elsewhere [16] that there is such a constraint on both bosonic and fermionic particles, using the two entangled particles that in a two double-slit device.

If we consider $y = (y_1+y_2)/2$ to be the vertical coordinate of the centre of mass of the two particles, then we can write

$$\begin{aligned}\dot{y} &= (\dot{y}_1+\dot{y}_2)/2 \\ &= \frac{\hbar}{2\sqrt{2}m} Im\frac{1}{\psi}(-\frac{y_1+y_2}{2\sigma_0\sigma_t})(\psi_{A_1}\psi_{B_2}\pm\psi_{A_2}\psi_{B_1}) \\ &= \frac{(\hbar/2m\sigma_0^2)^2}{1+(\hbar/2m\sigma_0^2)^2t^2}yt.\end{aligned} \tag{20}$$

Solving the differential equation, we get the path of the y coordinate of the centre of mass

$$y = y_0\sqrt{1+(\hbar/2m\sigma_0^2)^2t^2}. \tag{21}$$

Using equation (21) and similar to what is done in ref. [16], one can obtain the quantum potential for the centre of mass motion (Q_{cm})

$$Q_{cm} = \frac{my_0^4}{2y^2}(\frac{\hbar}{2m\sigma_0^2})^2 = \frac{1}{2}my_0^2\frac{(\hbar/2m\sigma_0^2)^2}{1+(\hbar t/2m\sigma_0^2)^2}. \tag{22}$$

If at $t = 0$ the centre of mass of the system is exactly on the x-axis, then $y_0 = 0$, and centre of mass of the system will always remain on the x-axis. In addition, the quantum potential for the centre of mass of the two particles is zero at all times. Thus, we have $y_1(t) = -y_2(t)$ and the two particles, in both the bosonic and fermionic case, will be detected at points symmetric with respect to the x-axis. This differs from the prediction of SQM, as the probability relation (11) shows. SQM predicts that probability of asymmetrical detection of pairs of particles can be different from zero in opposition to BQM's symmetrical prediction. Furthermore, according to SQM's prediction, the probability of finding two particles at one side of the x-axis can be non-zero while it is shown that BQM forbids such events, provided that $y_0 = 0$. Figure 1 shows one of the typical inconsistencies which can be considered at the individual level. Based on

BQM, bosonic and fermionic particles have the same results, but, we know that if one bosonic particle pass through upper (lower) slit, it must detected on the upper (lower) side on the S_2 screen, due to relations (19). Instead, there is no such constraint on fermionic particles.

We can assume that two particles are correlated so that in spite of their position distribution, y_0 can be always considered to be on the x-axis. However, one can argue that, it is necessary to consider a position distribution for y_0 based on the quantum equilibrium assumption, that is, $\triangle y_0 \neq 0$, but we can have $\langle y_0 \rangle = 0$. Therefore, not only symmetrical detection of two particles is violated, but also two particles can be found at one side of the x-axis on S_2 screen, because the majority of pairs can not be simultaneously on the x-axis [15]. One can solve these problems if we adjust $\triangle y_0$ to be very small and $\hbar t/2m\sigma_0^2 \ll 1$. We assume that, to maintain symmetrical detection about the x-axis with reasonable approximation, the centre of mass variation from the x-axis must be smaller than the distance between any two neighbouring maxima on the screen S_2, that is

$$y \ll \frac{\lambda D}{2Y} \simeq \frac{\pi \hbar t}{Ym}, \tag{23}$$

where λ is the de Broglie wavelength. For conditions $\hbar t/2m\sigma_0^2 \ll 1$, $Y \sim \sigma_0$ and using equation (21), one can obtain

$$y_0 \ll \frac{\pi \hbar t}{Ym} \sim \sigma_0. \tag{24}$$

Therefore, if we use a source with $\triangle y_0 \ll \sigma_0$, we will obtain $y \simeq y_0 \ll \sigma_0$ for each individual observation, and our symmetrical detection can be maintained with good approximation. In this case, we only lose our information about the trajectory of bosonic particles. It is evident that, if one consider $\triangle y_0 \sim \sigma_0$, as was done in [15], the incompatibility between the two theories will be disappeared. But, we believe that, instead of the usual one-particle two-slit experiment with $\triangle y_0 \sim \sigma_0$, our two-particle system provides a new situation in which we can adjust y_0 independent of σ_0, so that

$$y_0 = \frac{1}{2}(y_1 + y_2)_{t=0} \ll \sigma_0. \tag{25}$$

Although it is obvious that $(\triangle y_1)_{t=0} = (\triangle y_2)_{t=0} \sim \sigma_0$, but the position constraint on the two particles in the source S_1 makes them always satisfy equation (25), which is not feasible in the one-particle two-slit devices.

Now, one can compare the results of SQM and BQM at the ensemble level. To do this, we consider an ensemble of pairs of particles that have arrived at the detection screen S_2 at different times t_i. It is well known that, in order to ensure the compatibility between SQM and BQM for ensemble of particles, Bohm added a further postulate to his three basic and consistent postulates [1-3]. Based on this further postulate, the probability that a particle in the ensemble lies between $\overrightarrow{x}$ and $\overrightarrow{x} + d\overrightarrow{x}$, at time t, is given by

$$P(\overrightarrow{x}, t) = R^2(\overrightarrow{x}, t). \tag{26}$$

Thus, the joint probability of simultaneous detection for all pairs of particles of the ensemble arriving at S_2 is

$$P_{12} = \lim_{N\to\infty} \sum_{i=1}^{N} R^2(y_1(t_i), -y_1(t_i), t_i) \equiv \int_{-\infty}^{+\infty} dy_1 \int_{-\infty}^{+\infty} dy_2 |\psi(y_1, y_2, t)|^2 = 1, \tag{27}$$

where, every term in the sum shows only one pair arriving on the screen S_2 at the symmetrical points about the x-axis at time t_i with intensity of R^2. If all times of t_i in the sum is taken to be t, the summation on i can be converted to an integral over all paths that cross the screen S_2 at that time so that we obtain an interference pattern. Then, one can consider the joint probability of detecting two particles at two arbitrary points y_M and y_N which can belong to different pairs

$$P_{12}(y_M, y_N) = \int_{y_M}^{y_M+\Delta} dy_1 \int_{y_N}^{y_N+\Delta} dy_2 |\psi(y_1, y_2, t)|^2, \tag{28}$$

which is similar to the prediction of SQM, but obtained in a Bohmian way, as was shown by Ghose [12]. Thus, it appears that for such conditions, the possibility of distinguishing the two theories at the statistical level is impossible, as was expected [1-3, 9, 18].

Here, to show equivalence of the two theories, we have assumed for simplicity that $y_0 = 0$. If one consider $y_0 \neq 0$ or $\triangle y_0 \neq 0$, the equivalence of the two theories is maintained, as it is argued by Marchildon [15]. But, using this special case, we show that assumption of $y_0 = 0$ is consistent with statistical results of SQM and in consequence, finding of such a source may not be impossible.

3.2 Predictions for unentangled wave function

In this subsection we study the unentangled wave function (10), using BQM. Based on equations (14) and (15), Bohmian velocities of particle 1 and 2 can be obtained as

$$\begin{aligned} \dot{y}_1 &= N\frac{\hbar}{m} Im \frac{1}{(\psi_{A_1} + \psi_{B_1})}([\frac{-2(y_1 - Y - u_y t)}{4\sigma_0\sigma_t} + ik_y]\psi_{A_1} \\ &+ [\frac{-2(y_1 + Y + u_y t)}{4\sigma_0\sigma_t} - ik_y]\psi_{B_1}, \end{aligned} \tag{29}$$

$$\begin{aligned} \dot{y}_2 &= N\frac{\hbar}{m} Im \frac{1}{(\psi_{A_2} + \psi_{B_2})}([\frac{-2(y_2 - Y - u_y t)}{4\sigma_0\sigma_t} + ik_y]\psi_{A_2} \\ &+ [\frac{-2(y_2 + Y + u_y t)}{4\sigma_0\sigma_t} - ik_y]\psi_{B_2}, \end{aligned} \tag{30}$$

Thus, as we expected, the speed of each particle is independent of the other. Using this relations and equations (18), we have

$$\begin{aligned} \dot{y}_1(x_1, y_1; t) = -\dot{y}_1(x_1, -y_1; t), \\ \dot{y}_2(x_2, y_2; t) = -\dot{y}_2(x_2, -y_2; t). \end{aligned} \tag{31}$$

This implies that the y component of the velocity of each particle would vanish on the x-axis. Although these relations are the same relations that were obtained for the entangled wave function, but here, we have an advantage: none of the particles can cross the x-axis nor are tangent to it, independent of the other particle position. This property can be applied to obtain BQM's predictions incompatible with SQM.

To see this incompatibility, we consider a special detection process on the S_2 screen-something we call selective detection. In the selective detection, we register only those pair of particles which are detected on the two sides of the x-axis, simultaneously. That is, we eliminate the cases of detecting only one particle or when both particles of the pair are detected on the upper or lower part of the x-axis on the screen. Under these conditions, we can obtain the equation of motion of the centre of mass in the y-direction. Using equation (29) and (30), one can show that

$$\dot{y} = \frac{(\hbar/2m\sigma_0^2)^2 t(y_1+y_2)/2}{1+(\hbar/2m\sigma_0^2)^2t^2} + N\frac{\hbar}{2m}Im\frac{1}{\psi}(\frac{Y+u_yt}{\sigma_0\sigma_t}+2ik_y)(\psi_{A_1}\psi_{A_2}-\psi_{B_1}\psi_{B_2}). \quad (32)$$

Based on our selective detection and relations (31), each particle passes through one of the slits, A or B. In the other words, particles detected on the upper (lower) side of the x-axis, must pass through the slit $A(B)$. Thus, we can write

$$\begin{aligned} \psi_{A_1}\psi_{A_2} &\approx 0, \\ \psi_{B_1}\psi_{B_2} &\approx 0, \end{aligned} \quad (33)$$

and the equation of motion for the y coordinate of the centre of mass is reduced to

$$\dot{y} \simeq \frac{(\hbar/2m\sigma_0^2)^2}{1+(\hbar/2m\sigma_0^2)^2t^2}yt. \quad (34)$$

Similar to the past, we have

$$y \simeq y_0\sqrt{1+(\hbar/2m\sigma_0^2)^2t^2}. \quad (35)$$

It is clear that for $y_0 = 0$ or $\langle y_0\rangle = 0$, with $\triangle y_0 \ll \sigma_0$ and $\hbar t/2m\sigma_0^2 \ll 1$, disagreement between the two theories at the individual level is the same past, because based on BQM, we have symmetrical detection of two particles simultaneously, instead of SQM's prediction.

Now, consider conditions under which $y_0 \neq 0$ and $\hbar t/2m\sigma_0^2 \gg 1$. Then, the x-axis will not be an axis of symmetry and we have a new point on the S_2 screen along the y-axis around which all pairs of particles will be detected symmetrically. Thus, based on BQM, that is relations (31) and (35), there will be a length

$$L = 2y \simeq \frac{\hbar t y_0}{m\sigma_0^2}, \quad (36)$$

on the screen S_2 where no particle is recorded, as shown in Figure 2. On the other hand, based on SQM we have two alternatives:
i) The joint probability relation (11) is still valid and there is only a reduction in the intensity throughout the screen S_2.
ii) SQM is silent about our selective detection.
In the first case, there is disagreement between the predictions of SQM and BQM and in the second case, BQM has a better predictive power, even at the statistical level.

Instead of the last result, the adjustment of y_0 at a well defined position produces disagreement between the two theories at the statistical level. Thus, one can argue that, since y_0 must be distributed ($\triangle y_0 \neq 0$), the empty interval is not observed and statistical results of the two theories are the same. Response to this objection can be considered as before. Assume that $\triangle y_0 \neq 0$ and $\langle y_0\rangle$ is positive. Then we have a relative empty interval with low intensity of particles that has a length

$$L \sim 2\langle y\rangle \simeq \frac{\hbar t}{m\sigma_0^2}\langle y_0\rangle. \quad (37)$$

If one considers $\triangle y \ll L$, the empty interval will clearly appear. It is obvious that, $\triangle y \ll L$ corresponds to $\triangle y_0 \ll \langle y_0\rangle \sim \sigma_0$. Therefore, based on BQM, a very small change in the position of the source S_1 toward positive (negative) y-direction produces a region with very low intensity in the interference pattern above (below) the x-axis which cannot be predicted by SQM.

In addition, based on our factorizable wave function (10), one can object that, each particle simply follows one of the single-particle two-slit trajectories and is quite independent of the other particle and in consequence, both SQM and BQM yield the same result. But, one can see that this objection is unfounded for our specified conditions. If we study the interference pattern without using selective detection, we must obtain the same result for the two theories, as has been claimed. But, using selective detection, it is clear that not only the two theories have not the same statistical predictions, but also BQM clarifies and illuminates SQM, as Durr et al. [18] said:"note that by selectively forgetting results we can dramatically alter the statistics of those that we have not forgotten. This is a striking illustration of the way in which Bohmian mechanics does not merely agree with the quantum formalism, but, eliminating ambiguities, clarifies, and sharpens it.". In our selective detection, we have forgotten single-particles and two-particles detected at the one side of x-axis, simultaneously, on the screen S_2. In ref. [3] the trajectories for the Gaussian slits with a Gaussian distribution of initial positions at each slit is shown. If we assume that $0 < y_0 < \sigma_0$, the lower trajectories at the upper slit will present single-particle trajectories. In addition, pairs of particles which pass through one of the slits can sit any where on the screen S_2, according to their initial conditions. Since we have registered only those pair of particles which arrive on the two sides of the x-axis, simultaneously, and the other detected particles–including single particles and pairs of particles detected at one side of the x-axis–is forgotten, our obtained statistical disagreement is a clear result. In addition, we have shown elsewhere [17] that, there is statistical disagreement between the two theories for a new two-particle system described with an entangled wave function, using a different selective detection and without any deviation of the source from the x-axis. Therefore, it seems that performing such experiments provide observable differences between the two theories, particularly at the statistical level.

4 Conclusion

In this article, we have suggested a thought experiment to distinguish between standard and Bohmian quantum mechanics. The suggested experiment contains a usual two-slit interferometer with a special source which emits two identical particles. If such sources are found, the thought experiment will convert to a practicable experiment. We have shown that our two-particle system can be described with two kinds of wave functions, entangled and unentangled wave functions. For the entangled wave function, we have obtained some disagreement between SQM and BQM at the individual level. But, it is shown that, the two theories predict the same statistical results, as was expected. For the unentangled wave function, the predictions of the two theories could be also different at the individual level, similar to the last wave function. Again, the results of the two theories were the same at the ensemble level. However, we have shown that, selective detection can dramatically alter the interference pattern so that not only the statistical results of BQM do not agree with those of SQM, but BQM can also increase our power prediction. Therefore, it seems that, our suggested thought experiment can decide between the standard and the Bohmian quantum mechanics.

References

[1] D. Bohm, Part I, Phys. Rev. 85 (1952) 166; Part II, 85 (1952) 180.

[2] D. Bohm, B.J. Hiley, The Undivided Universe, Routledge, London, 1993.

[3] P.R. Holland, The Quantum Theory of Motion, Cambridge University Press, Cambridge, 1993.

[4] J.T. Cushing, Quantum Mechanics: Historical Contingency and the Copenhagen Hegemony, The University of Chicago Press, Ltd., London, 1994.

[5] M. Abolhasani, M. Golshani, Phys. Rev. A 62 (2000) 12106.

[6] B.G. Englert, M.O. Scully, G. Sussman, H. Walther, Z. Naturforsch. 47a (1992) 1175.

[7] M.O. Scully, Phys. Scripta, T 76 (1998) 41.

[8] C. Dewdney, L. Hardy, E.J. Squires, Phys. Lett. A 184 (1993) 6.

[9] B.J. Hiley, R.E. Callaghan, O.J.E. Maroney, Quantum Trajectories, Real, Surreal or an Approximation to a Deeper Process?, quant-ph/0010020.

[10] J.P. Vigier, Phys. Lett. A 270 (2000) 221.

[11] A.J. Leggett, Found. Phys. 29 (1999) 445.

[12] P. Ghose, Incompatibility of the de Broglie-Bohm Theory with Quantum Mechanics, quant-ph/0001024; An Experiment to Distinguish Between de Broglie-Bohm and Standard Quantum Mechanics, quant-ph/0003037.

[13] M. Golshani, O. Akhavan, A two-slit experiment which distinguishes between the standard and Bohmian quantum mechanics, quant-ph/0009040.

[14] L. Marchildon, No contradictions between Bohmian and quantum mechanics, quant-ph/0007068.

[15] L. Marchildon, On Bohmian trajectories in two-particle interfrence devices, quant-ph/0101132.

[16] P. Ghose, Comments on"On Bohm trajectories in two-particle interference devices" by L. Marchildon, quant-ph/0102131.

[17] M. Golshani, O. Akhavan, Bohmian prediction about a two-double slit experiment and its disagreement with the standard quantum mechanics, quant-ph/0103101.

[18] D. Durr, S. Goldstein, N. Zanghi, J. Stat. Phys. 67 (1992) 843.

Figure Caption

Figure 1. A two-slit experiment in which two identical particles are emitted from the source S_1, then they pass through slits A, B, and finally they are detected on the screen S_2, simultaneously. We assume that $y_0 = 0$ or $\langle y_0 \rangle = 0$ under $\triangle y_0 \ll \sigma_0$ and $\hbar t/2m\sigma_0^2 \ll 1$ conditions. It is clear that dashed lines are not real trajectories.

Figure 2. The same two-slit experiment in which $\langle y_0 \rangle > 0$, $\triangle y_0 \ll \sigma_0$, $\hbar t/2m\sigma_0^2 \gg 1$, and selective detection is considered. All detections are symmetric on the two sides of y_{cm} on the screen S_2. Thus, L shows the empty interval in the final observed pattern. Dotted dashed lines are not real trajectories.